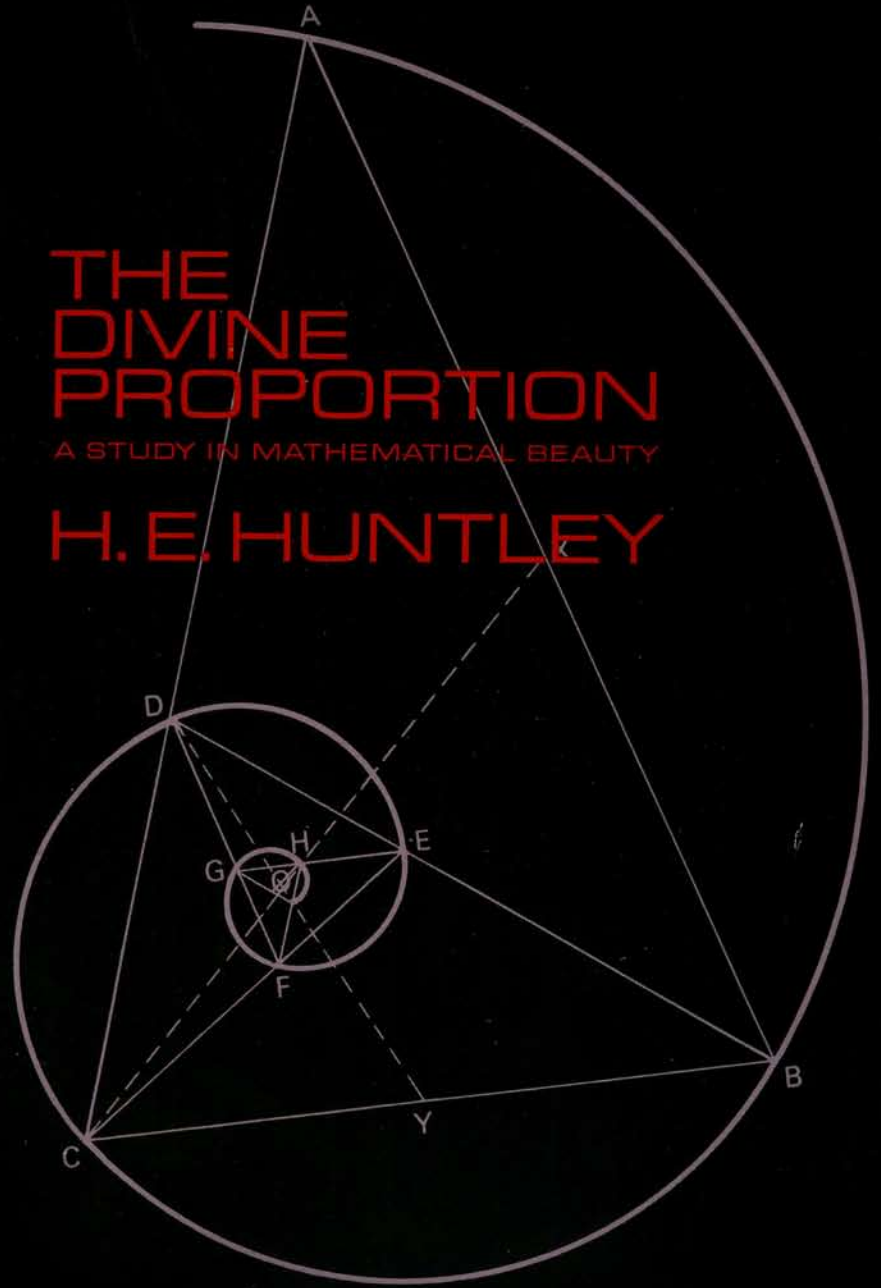


THE DIVINE PROPORTION

A STUDY IN MATHEMATICAL BEAUTY

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C H A P T E R I

The Texture of Beauty

Before launching out on our main topic, beauty in mathematics, it will be worth while to convince ourselves that the effort required to learn to appreciate aesthetic values is justified by the pleasure it offers. The conviction is born of experience; and we shall soon discover that it is shared by many of the world's wisest men. An example of ancient standing, written before the Christian era (*Ecclesiasticus* 43, vv. 11-12), magnifies one of the most familiar of beautiful objects:

Look upon the rainbow, and praise him that made it; very beautiful it is in the brightness thereof. It compasseth the heaven about with a glorious circle, and the hands of the most High have bended it.

A further quotation relating to the same example of beauty will serve to underline one of the important lessons of these chapters. For aesthetic appreciation there are two requirements: the first is *given*, the second *acquired*. The first is from nature—by inheritance; the second from nurture—by education.

If the poet sees beauty in a rainbow—

My heart leaps up when I behold
A rainbow in the sky...

—so does the physicist in the laws governing its manifestation:

His heart leaps up, too, as he discovers how the light of day is reflected, chromatically refracted, reflected again and dispersed by gently falling water spheres into a thousand hues, conforming the while to lovely theorems of mathematics so simple in some aspects that the schoolboy may understand, so complex in others as to defy analysis.¹

The surface beauty of the rainbow—"very beautiful it is in the brightness thereof"—is appreciated by all men: it is *given*. But the buried beauty, uncovered by the industrious researches of the physicist, is understood only by the scientifically literate. It is *acquired*: education is essential.

DEFINITIONS

It is difficult to define beauty, as we shall see; but there is much impressive testimony to the importance of the emotions that beauty calls forth. Mohammed said:

If I had only two loaves of bread, I would barter one to nourish my soul.

A more modern witness, Richard Jefferies, wrote:

The hours when we are absorbed by beauty are the only hours when we really live. . . . These are the only hours that absorb the soul and fill it with beauty. This is real life, and all else is illusion, or mere endurance.

Beauty is a word which has defied the efforts of philosophers to define in a way that commands general agreement. Yet it does not need a philosopher's wisdom to utter a few meaningful words about it. One incontrovertible statement might be: beauty arouses emotion. This, being sufficiently indefinite, needs no qualification. The case would be different if we said, "This lovely artifact always arouses pleasurable emotion in everyone who sees it," for we know that some people, confronted by beauty which moves others, are entirely unresponsive to it. This appears to justify the familiar aphorism: "Beauty lies in the eye of the beholder." Whether this be true or not, it is certain that no philosopher, however erudite, can contradict me when I say in sincerity, concerning some experience,

“For me, that is beautiful.” But whether beauty is subjective or objective or both is an unresolved metaphysical problem.

According to the *Shorter Oxford English Dictionary* beauty is:

that quality or combination of qualities which affords keen pleasure to the senses, especially that of sight, or which charms the intellectual or moral faculties.

Only a part of this wide definition concerns us here. We are not interested at the moment in, for example, “the beauty of holiness” which “charms” the moral faculties. Our interest lies in the combination of qualities which charms the intellect. “Combination of qualities” reminds us that the experience of beauty is not a simple, but a complex experience. In mathematics it may be compounded of surprise, wonder, awe, or of realised expectation, resolved perplexity, a sense of unplumbed depths and mystery; or of economy of the means to an impressive result. When the mathematician refers to beauty in mathematics, we infer that he has had experience of some or all of these qualities.

Before we turn to the consideration of particular types of beauty, it is profitable to think of it in a wider context.

EVOLUTION OF AESTHETIC FACULTY

Taking a teleological view-point, we might begin by asking whether the universal human thirst for beauty serves a useful purpose. Physical hunger and thirst ensure our bodily survival. The sex drive takes care of the survival of the race. Fear has survival value. But—to put the question crudely—what is beauty for? What personal or evolutionary end is met by the appreciation of a rainbow, a flower or a symphony? At first sight, none. Why, if I have two loaves, should I “sell one and buy a lily”? Many of our appetites have been developed in the course of human evolution for a utilitarian purpose in the material environment of our mundane existence. Does this suggest a realm of another natural order? That it points to a definitive view of the nature of the human psyche is a conclusion which seems unavoidable. Before we develop this, let us remind ourselves of how some philosophers, both ancient and modern, have regarded beauty.

Plato, in the *Symposium*, has much to say about progress from

aesthetic appreciation to the enjoyment of “absolute beauty.” He recounts an inspired speech by Socrates in a dramatic dialogue at the “Dinner Party.” Socrates modestly attributes his views to his “instructress”—a woman from Mantinea, called Diotima. The following excerpts are relevant to our subject:

The man who would apply himself to this goal must begin, when he is young, by applying himself to the contemplation of physical beauty. . . . The next stage is for him to reckon beauty of soul more valuable than beauty of body. . . . From morals he must be directed to the sciences and contemplate their beauty also. . . . [The man] who has directed his thoughts towards examples of beauty in due and orderly succession will suddenly have revealed to him as he approaches the end of his initiation a beauty whose nature is marvellous indeed, the final goal, Socrates, of all his previous efforts. This beauty is first of all external; it neither comes into being nor passes away; next, it is not beautiful in part and ugly in part, nor beautiful at one time and ugly at another. . . . He will see it as absolute, existing alone with itself, unique, external, and all other beautiful things as partaking of it. . . .

This above all others, my dear Socrates, (the woman from Mantinea continued) is the region where a man’s life should be spent, in the contemplation of absolute beauty. Once you have seen that, you will not value it in terms of gold or rich clothing or the beauty of boys and young men. . . . What may we suppose to be the felicity of the man who sees absolute beauty in its essence, pure and unalloyed, who, instead of a beauty tainted by human flesh and colour and a mass of perishable rubbish, is able to apprehend divine beauty where it exists apart and alone? Do you think that it will be a poor life that a man leads who has his gaze fixed in that direction, who contemplates absolute beauty with the appropriate faculty and is in constant union with it?

Turning from an ancient to a modern philosopher, we may consider the views of the Italian philosopher, Benedetto Croce. His position is that beauty is an attribute of that which expresses feeling. Music, as Plato recognized, expresses human emotion very vividly; it is *lento*, *vivace*, *con brio*, etc. Beauty is seen in colors that are gay or somber. And there is the beauty of scenery:

Bright robes of gold the fields adorn,
The hills with joy are ringing,
The valleys stand so thick with corn
That even they are singing.

Clouds are lonely or angry; the morn is smiling; the oak is majestic; a mathematical theorem is elegant, its proof neat.

Wordsworth said of poetry that it was "emotion recollected in tranquillity."

Following Croce, then, we may take it that the aesthetic experience supervenes when some material or mental entity, to which for that reason we attribute "beauty," stimulates pleasurable emotion.

Now emotions are regarded by psychologists as activities of the unconscious mind, so that the aesthetic experience is the resuscitation of subliminal emotions, and beauty is the power to evoke these emotions. This takes us into deep waters and we will postpone a discussion of the function of the unconscious in mathematical studies to a later chapter.

UNITY IN VARIETY

For a modern view of the nature of beauty we may turn again to J. Bronowski:

When Coleridge tried to define beauty, he returned always to one deep thought; beauty, he said, is unity in variety! Science is nothing else than the search to discover unity in the wild variety of nature,—or, more exactly, in the variety of our experience. Poetry, painting, the arts are the same search, in Coleridge's phrase, for unity in variety.²

A WORKING HYPOTHESIS

A student who aspires to gain an insight into a philosophy of the beauty that is latent in mathematics should fortify himself with some form of working hypothesis concerning beauty in a wider context, with the help of modern views on the nature of the human psyche as developed by Sigmund Freud, Carl Jung and other psychologists. Such an hypothesis will not have the status of a theory. While it will, of course, require modification and amplification as new knowledge is gained, it is not thereby invalidated as a viable frame of reference, of which the function is to maintain a logical sequence among numerous data.

Fundamental to such an hypothesis is the recognition that the aesthetic experience is an emotional, rather than a rational mental

activity. Merely to state this basic fact is to realize that we shall not make much progress in understanding without admitting the relevance of what has been called "the greatest discovery of the nineteenth century"—the *subconscious mind*. Though psychologists have found this topic a fertile source of differences of opinion, they are agreed concerning its importance in interpreting mental activity. It is invoked to explain such phenomena as hypnotic trance, dreams, narcosis, dual personality, mental disorders and much more. Its value for our present purpose is that it provides a clue to the understanding of aesthetic feeling.

STRUCTURE OF THE PSYCHE

Psychologists often use the wider terms *psyche* and *psychic* in place of mind and mental, which are normally applied to the conscious mind only. Psychic activity is no less real than physiological activity: the psyche has its own structure and is governed by its own laws. A montage of the psyche as seen by the pioneer psychoanalyst Carl Jung would cover four main concepts:

1. The *conscious mind*, or "surface" mind, the seat of conscious mental activity.
2. The *preconscious*, sometimes pictured as forming the periphery of the conscious mind, sometimes as the stratum below the surface mind. Our memories of recent events, now removed from the focus of attention, are stored herein. From here they may be voluntarily recalled—recollected.
3. The *subconscious* or *personal unconscious*. The conscious mind, according to Jung, is

based upon and results from an unconscious psyche which is prior to consciousness, and continues to function together with, or despite consciousness.³

The dipstick of introspection cannot plumb this layer of the psyche. Unconscious activity is only exceptionally recognized by the individual, despite the fact that unconscious motivation is one of the prime facts of life. In the unconscious are stored countless forgotten memories which, while they cannot be recalled at will, are nevertheless made manifest in dreams, in hypnotic trance and through other means.

4. The *collective unconscious*, according to Jung, forms a lower stratum of the psyche than the personal unconscious. It is the source of instinctive behaviour, an instinct being defined as "an impulse to action without conscious motivation." Instinctive behavior is inherited: it is determined by the history of the race. So are what Jung calls "primordial images" or "archetypes," which were formed at low mental levels during the tens of thousands of years of the evolutionary history of primitive man, our remote human ancestors, by the constant recurrence of universal emotional experiences common to all, e.g., the alternation of day and night, seasonal changes, hunger and thirst, flight from danger, the mountains and the oceans, storm and tempest, the sanctuary of hearth and home.

As a mnemonic, the structure of the psyche may be compared to an ocean island. The land above the water surface represents the conscious mind, the area uncovered at low tide depicts the preconscious; the vast, hidden mass of rock below the ocean represents the unconscious which rises from an ocean bed standing for the collective unconscious. The scientists halt here, but the theologians (notably Tillich) speak of the deepest level of all, which undergirds the ocean bed, as "the ground of our being," and equate it with God.

EMOTIONAL ACTIVITY

If we may assume that the evolution of psychic potentialities through geological ages has run parallel to the development of the nervous system and the brain, it would appear that, historically, emotional life which we share with the higher animals must precede intellectual development and be associated with the primitive parts of the nervous system. Incidentally, this also controls the visceral activities of the body and that is why a public performer afflicted with "nerves" sometimes has cause to observe the connection between his emotion and the activity of his intestines! We must accordingly conclude that the personal unconscious, as well as the collective unconscious, is the arena of the emotions as well as the storehouse of emotive memory complexes.

Now, since the structure of the nervous system is inherited, it

is not unreasonable to suppose that the physiological conditions favorable to the animation of primordial emotions of the collective unconscious are also handed down from generation to generation. It is accordingly natural to postulate that the tendency of the psyche to make certain broad aesthetic judgments relating to the common human environment is inherited. H. J. Eysenck refers to the hypothesis, based on experiments, that

there exists some property of the nervous system which determines aesthetic judgments, a property which is biologically derived. . . . One deduction, for instance, might be that this ability (aesthetic judgment) should be very strongly determined by heredity; there is already some evidence for this point of view. . . .⁴

AESTHETICS

Let us now turn from general considerations to the particular case of the emotion generated by the interaction between an object of beauty and an observer—the aesthetic feeling. If the foregoing sketch of a working hypothesis is on the right lines, then the aesthetic experience consists in the levitation from the unconscious to the surface mind of a memory complex activated by an association mechanism sequential to the visual or aural contemplation of the beautiful object. It is not difficult to guess the nature of these hidden memory complexes: they arise from the immemorial terrestrial environment of man. The complexity of this defies analysis, but it will make our meaning clear if we point to a few specimen experiences which have been familiar to both men and animals for a million years: (i) color contrasts, (ii) the gravitational field, (iii) bird song, human conversation and vocal music.

i. Our pleasure in color is shared with some of the vertebrates. Dr. W. H. Thorpe, describing the Bower birds of Australia and New Guinea, states that they build bowers for courtship with

brightly coloured fruits or flowers which are not eaten but left for display and replaced when they wither. . . . They stick to a particular colour scheme. Thus, a bird using blue flowers will throw away a yellow flower inserted by the experimenter, while a bird using yellow flowers will not tolerate a blue one.⁵

Dr. Thorpe quotes Robert Bridges: "Verily it may well be that

sense of beauty came to those primitiv bipeds earlier than to man." In that case we should not be surprised if mankind's collective unconscious, carrying such an inheritance from the lower creation, is deeply stirred at the sight of flowers, a colorful sunset or a rainbow, rousing the conscious mind, in due sequence, to the aesthetic response.

ii. Apart from its colors, the gentle curve of the rainbow would, through association, stimulate memories stored in the collective unconscious by the ever-present phenomena of the earth's gravitational field—the lovely parabolic path of a flying stone, or spear or arrow, of the water drops of a fountain or cascade. All parabolas have the same unique shape, of which the mighty circular arc of the rainbow is reminiscent.

iii. Similar considerations apply to the beauty of music. Dr. Thorpe remarks that

... it is perhaps plausible that the intervals which are acceptable to the human ear, as normal and natural for music, are in fact those intervals which were first offered to the ancestors of man by bird song. Other animals do not have much in the way of song; but the fundamental intervals of human and bird song are the same; and highly developed bird song was audible at man's first appearance in time. Since man always had bird song all around, impinging on his ears, is it not reasonable to suppose that he developed a musical signal system by imitating the birds?⁶

MUSIC

Music is the language of the unconscious mind par excellence. As we shall argue in chapter VI, primordial racial memories are brought to the surface more readily by music than by natural scenery or any other art; it seems to be possible to relate familiar features of music to archaic experiences of humanity.

It is music that provides the strongest support for our thesis that aesthetic experience consists in the interaction between the universal primordial images buried in the unconscious and an external artifact or natural object which we call beautiful.

The incomparable power of music to move a listener to the depths of his being is well-known; it will, on occasion, bring him to tears. What is the explanation of the power of this stimulus

which is unparalleled in the other arts? If our thesis is tenable it must be that music is for some reason an unusually effective agent for bringing to the surface archaic images and memories stored in the unconscious. As Jung remarks (see p. 77), "The man who speaks with primordial images speaks with a thousand tongues. . . . That is the secret of effective art." Now musical expression can stimulate archaic emotional experiences very effectively—fear by *agitato*, mourning by *molto legato*, excitement by *prestissimo*, sanctuary by *rallentando* succeeded by the tonic or *home* note, and in similar ways. These universal emotionally charged experiences become effective when they are raised from the deep unconscious to the surface mind, and it happens that music, unlike any of the other arts, provides precise and powerful means of effecting this transfer.

When a hypnotist induces a trance in a suitable subject, he finds a simple, potent method in making rhythmic passes with his hands in front of the subject's eyes. This induces a light trance and increases the subject's suggestibility. He will then be ready to accept almost any suggestion—even an absurd one—that the hypnotist makes. In particular, as Eysenck points out,

Under hypnosis a person can remember things which in the normal state he would be quite unable to recall. In fact, it is suggested that under hypnosis a person can be "regressed" to an earlier age and that in this condition he will experience again the events which were happening at that time and the emotions which they evoked in him.⁷

In the case of music, the rhythmic movements of the hypnotist's hands are replaced by those of the conductor's baton which is amplified by the rhythm of the music. When the "beat" is very strong, as in dance music among primitive tribes, the hearers become entranced. Even under the more familiar conditions of an audience listening to Western music, it may be presumed that most if not all listeners are under the influence of a very light trance which increases their suggestibility and facilitates the levitation of buried archaic images to the conscious mind.

It is along such general lines that we may look for the "secret of the effective art" of music and an explanation of the aesthetic pleasure resulting from it.

And now let us return to the consideration of the question of

whether beauty serves a purpose in the scheme of creation. We have already seen that it appears to serve no utilitarian end. Many of our instincts and associated emotions have been evolved to ensure our bodily survival, but the emotion aroused by a physical object such as a cloud or a flower, or by a mental image such as an elegant mathematical theorem, has no such objective. The answer to the question which we posed in crude terms: "What is beauty for?" appears to be elusive. So much is this so that one is inclined to doubt whether it has any purpose and to dismiss the matter by asking impatiently, "Must all things have a *raison d'être*? Is not a thing of beauty a joy for ever, and, so far from being a means to an end, an end in itself?"

AESTHETIC PLEASURE UNIVERSAL

It seems to me to be important that we should have clear ideas in reference to this question, and I hope the reader, before he proceeds to the chapters which follow, will give careful consideration to the point of view now to be described. At first glance it may appear that the contemplation and appreciation of the beauty of, say, a mathematical theorem is an unimportant, even trivial, activity. On the contrary, it is, properly regarded, one of great significance. It would seem to be unlikely, *a priori*, that the whole human race should be endowed with the faculty to enjoy beauty unless it achieved some noble consummation. "Earth's crammed with heaven and every common bush aflame with God" to some purpose, surely? The power to appreciate beauty appears to be a human endowment and this suggests that we should seek its origin and its purpose in human nature—in that nature which distinguishes us from the animal creation. Thus, for an answer to our question, we are driven back to the explanation of our human nature given in *Genesis* 1, v. 26:

And God said, Let us make man in our image, after our likeness.

Here, I suggest, is the clue. Man is by nature a *creator*. After the likeness of his Maker, man is born to create: to fashion beauty, to originate new values. That is his supreme vocation. This truth awakens a resonant response deep within us, for we know that one of the most intense joys that the soul of man can experience is

happier = no fear? = love of beauty?

that of creative activity. Ask the artist. Ask the poet. Ask the scientist. Ask the inventor or my neighbor who grows prize roses. They all know the deep spiritual satisfaction associated with the moment of orgasm of creation.

CREATIVE ACTIVITY: EMPATHY

This deep joy has been thought by some to be the principal aim of education—more, the chief end of human life. In *The Education of the Whole Man*, L. P. Jacks writes:

What then is the vocation of the whole man? So far as I can make out, his vocation is to be a creator: and if you ask me, Creator of what?, I answer—creator of real values. . . . And if you ask me what motive can be appealed to, what driving power can be relied on, to bring out the creative element in men and women, there is only one answer I can give; but I give it without hesitation—the love of beauty, innate in everybody, but suppressed, smothered, thwarted in most of us. . . .⁸

This inborn love of beauty, our human heritage, *must* find expression if we are to be happy. If the hunger for beauty remains unsatisfied, the effects are seen in loss of physical and mental health, so deep is the need.

We now approach the final stage in the argument of this chapter. It underlines a truth which it is important that all students of mathematics should understand, but (it is to be feared) very few do. If it could be expressed in one word, that word would be *empathy*. The German equivalent is *Einfühlung*—"feeling into."

We have spoken of a common experience—the joy associated with any form of creative activity, which a man has as a consequence of his having been made in the image of his Creator. And we have interpreted the mystery of the nature and purpose of beauty by recalling the familiar fact that the inborn faculty of aesthetic appreciation constitutes the motive for the creation of objects of beauty. And now we have to meet the natural objection that many would raise: they have had no experience of creative activity. They have added nothing to the store of beauty, their own ideas have been neither new nor original. They have never known the luminous moment of inspiration which widened the bounds of knowledge. They can appreciate, but cannot create beauty.

sympathy / empathy / crying = raining / magic
wholeness

The answer to this objection can be stated briefly. The act of creation and the act of appreciation of beauty are not, in essence, distinguishable. This is true whether the lovely object is a work of art, a musical composition or a mathematical theorem. In the actual moment of appreciation (“*I see! Yes, indeed I see! How beautiful!*”), the beholder experiences those precise emotions which passed through the mind of the creator in his moment of creation. With the help of the artist he himself shares the joy of creation. This important fact has been expressed with characteristic clarity by J. Bronowski:

The discoveries of science, the works of art are explorations—more, are explosions, of a hidden likeness. The discoverer or the artist presents in them two aspects of nature and fuses them into one. This is the act of creation in which an original thought is born, and it is the same act in original science and original art. . . . [This view] alone gives a meaning to the act of appreciation; for the appreciator must see the movement, wake to the echo which was started in the creation of the work. In the moment of appreciation we live again the moment when the creator saw and held the hidden likeness. . . . We re-enact the creative act, and we ourselves make the discovery again. . . . The great poem and the deep theorem are new to every reader, and yet are his own experiences, because he himself re-creates them. They are the marks of unity in variety, and in the instant when the mind seizes this for itself, the heart misses a beat.⁹

This passage, which illuminates the meaning of *empathy*, should be understood by all who seek the aesthetic experience. In particular, the reader of the following pages, whether his interest is focused on the golden cuboid, or the dodecahedron, or the logarithmic spiral or the genealogy of the drone bee, should realize that, in the act of appreciation, he is re-enacting the creative act and, attracted by beauty, is experiencing himself the joy of creative activity. He is in fact, in Kepler’s phrase, “thinking God’s thoughts after Him.”