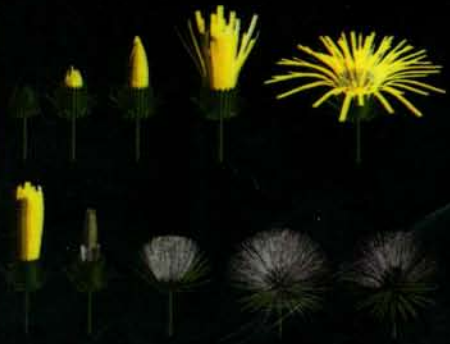
"A spectacular view of this new intellectual landscape."
–Vernor Vinge in *New Scientist*

would-be worlds

How Simulation Is Changing the Frontiers of Science



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sciences and mathematics, it seems highly unlikely that human creative capacity is subject to the rigid constraints of a Turing-type of computer. There are other models for computation besides that of Turing, and it may well be that the human mind is some type of super Turing machine, for example, a DNA computer. These matters are currently under investigation. But if the human mind is able to transcend the following of rules in its cognitive activity, then it would also not be subject to Gödelian limits of undecidability.

On this non-Gödelian note, let me conclude this examination of limits to scientific knowledge with a personal speculation about what would be needed for the world of physical phenomena to display the kind of logical undecidability seen in mathematics. Basically, my claim is that for this to happen nature would have to be either inconsistent or incomplete. What do these mathematical notions correspond to in the world of physical phenomena?

Consistency means that there are no true paradoxes in nature. Quantum mechanics notwithstanding, my view is that particles do not move to the left and to the right simultaneously, nor does water run uphill and downhill at the same time. Every time we have encountered what appeared to be such a paradox, as with the redshift of quasars or the seemingly slow rate of expansion of the early universe, subsequent investigation and theory has provided a resolution of it. So I will take it as an axiom of faith that nature is consistent.

Completeness of nature implies that a physical state cannot arise for no reason whatsoever; in short, there is a cause for every effect. Again, I can think of no incontrovertible counterexamples to this claim. So I will take it as a working hypothesis that nature is not only consistent, but also complete.

Putting these two assertions—consistency and completeness—together, I believe it is likely that there are no logical barriers to providing a scientific answer to any question we care to put to nature. Perhaps a tour of twentieth-century science is not so depressing after all!

Our leitmotif throughout this book has been the idea of using the computer as a tool for experimentation. The novelty resides in the relentless march of technology, a march that has finally provided us with computational capabilities allowing us to realistically hope to capture enough of the real world inside our programs to make these experiments meaningful. This raises the interesting question of what science

might have been like in the past if such technology had been available to thinkers like Aristotle, Newton, and Gauss. Let's speculate on this theme for a moment.

Computation *Über Alles*

Of the myriad stratagems I employ to avoid useful work, one of the most pleasurable is to speculate how scientists of earlier eras would have made use of modern computers and what effect it would have had on the science of their times—and ours. For example, I suspect that Newton's geometrically based arguments for particle motion would have remained essentially untouched by the hand of the machine (although Newton may well have used a relational database to trace out the various biblical lineages that seemed to have occupied most of his time). On the other hand, it's likely that Kepler would have discovered much more than his three laws of planetary motion, perhaps even anticipating Newton's equations, had he had access to a Sun or SGI workstation. There's no doubt in my mind that Gauss would have proved the Prime Number Theorem long before de la Vallée Poussin and Hadamard if he had been able to make use of packages like *Mathematica* or *Maple* to study the distribution of primes.

Recently, while surfing the Internet looking for some now long-forgotten item, I stumbled across a hyperlink to work of the great turn-of-the-century Scottish naturalist and polymath d'Arcy Wentworth Thompson. This serendipitous event underscored again the great advantage of search engines that are not *too* accurate or *too* efficient, because the Web page that I was directed to ended up being vastly more interesting than the material I was originally trying to track down. Anyway, as any self-respecting mathematical biologist knows by now, d'Arcy Thompson's greatest contribution to theoretical biology was in the last chapter of his magisterial treatise *On Growth and Form,* in which he put forth a theory of biological transformations whereby one might compare the shapes of living things. What popped up on my screen that morning was nothing less than an account of what d'Arcy Thompson would have done if he had had a computer on his worktable instead of a stack of graph paper, a ruler, and a compass.

D'Arcy Thompson held a professorial chair in St. Andrews and Dundee in Scotland for the amazing period of 64 years, a record for tenure unlikely ever to be broken. Although he would write more than 300 scientific articles and books, Thompson's reputation is based primarily upon his attempts to reduce biological phenomena to mathematics in *On Growth and Form.* There he claimed that much about animals and plants could be understood by the laws of physics, as mirrored in the structures and patterns of mathematics. By this, Thompson displayed his essentially anti-Darwinist beliefs, at least in the sense that he would most certainly have disagreed with Theodosius Dobzhansky's well-known remark that, "nothing in biology makes any sense except in the light of evolution." Clearly, Thompson felt that a *lot* of biology made perfectly good sense quite outside the bounds of the principle of natural selection. Besides his academic renown, Thompson seems also to have acquired a bit of a local reputation as a mild eccentric, and older folks in St. Andrews can still recall seeing him strolling about town with a parrot on his shoulder.

The novel idea Thompson set forth in *On Growth and Form* was to show how mathematical functions could be applied to the shapes of one organism to continuously transform it into other, physically similar organisms. A famous example from his book is shown in Figure 5.4, in which a continuous squeezing and stretching of a rectangular Cartesian grid transforms the fish species *Scarus sp.* on the left to the species *Pomacanthus* on the right. Thompson used this same idea to show how to alter pictures of baboon skulls into skulls of other primates, as well as demonstrating how corresponding bones like the shoulder blades are related in different species.

To a topologist, the fact that the fish species *Scarus sp.* can be continuously transformed into the species *Pomacanthus* is entirely

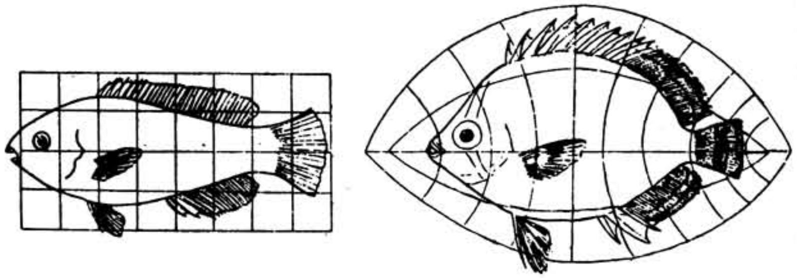


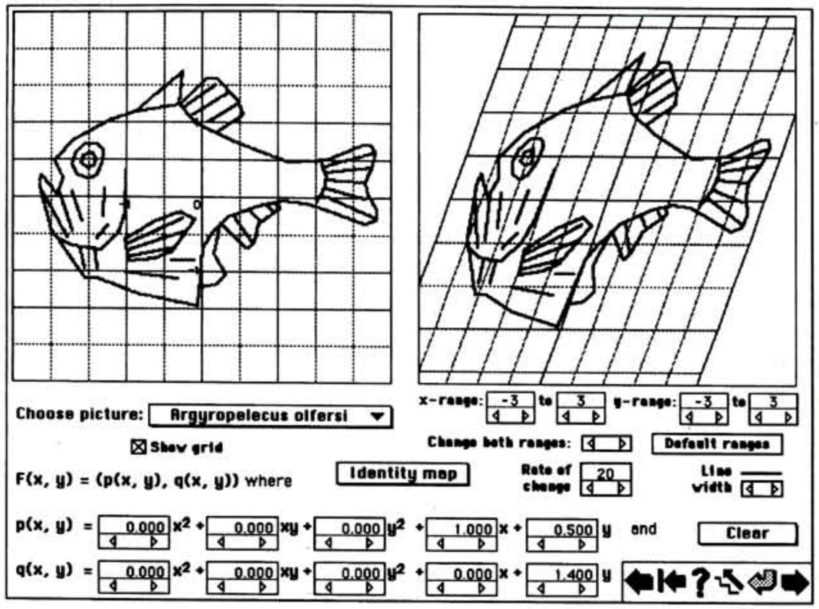
Figure 5.4 Two species of fish related by a continuous transformation.

unremarkable. This is because every organism is a closed surface, topologically speaking, and it has been known for many years what kinds of surfaces can and cannot be mapped to one another in various ways. Speaking very loosely, two surfaces are equivalent in this sense if and only if they have the same number of holes. Thus, considering the fact that just about all higher animals have the same number of holes for the digestive tract, ears, nostrils, and eyes, Thompson's work on biological transformations only confirms the topological fact that there exists a continuous transformation that will warp and twist the form of virtually any animal into any other one. So at first glance it doesn't seem as if there is too much meat in Thompson's idea. A world in which aardvarks and zebras are indistinguishable hardly seems to hold much promise for shedding light on the processes of embryology or evolution. But wait!

Simply knowing that there is *some* transformation deforming an aardvark into a zebra is a far different matter from knowing precisely *which* transformation(s) do the job. If there's anything to be said for d'Arcy's view that the basic processes of evolution and development can be understood mathematically via biological transformations, then that something resides in the precise nature of the transformation. It is to address exactly this point that I'm sure d'Arcy would have used a computer had such gadgets been available a century earlier. Fortunately, two of his successors on the faculty of the University of St. Andrews, John J. O'Connor and Edmund F. Robertson, took up this challenge and have created a program for studying the precise analytic form of these Thompson transformations.

The program written by O'Connor and Robertson allows users to alter pictures in real time by varying parameters in mathematical functions describing the transformations, thus seeing the picture change before their very eyes from, say, one fish to another. Figure 5.5 shows the program's user interface corresponding to another of d'Arcy Thompson's famous fish examples. The picture on the left, the species *Argyropelecus olfersi,* is mapped to the unknown species on the right by the quadratic transformation whose parameters are specified at the bottom of the screen. The reader will see from the figure that in this example the transformation only involves a linear stretching of the x and y axes, since the quadratic terms in the transformation are zero.

Symbolically, the program admits maps of the form $(x, y) \to \big(p(x, y), q(x, y)\big)$, which take a point (x, y) of the plane on the left to



Choose picture: Argyropelecus olfersi

☒ Show grid

x-range: -3 to 3 y-range: -3 to 3

Rate of change 20

$F(x, y) = (p(x, y), q(x, y))$ where

$p(x, y) = 0.000\,x^2 + 0.000\,xy + 0.000\,y^2 + 1.000\,x + 0.500\,y$ and

$q(x, y) = 0.000\,x^2 + 0.000\,xy + 0.000\,y^2 + 0.000\,x + 1.400\,y$

Figure 5.5 View of the user interface for the Thompson transformation program.

the point (p, q) in the plane on the right, where p and q are quadratic functions in the two variables x and y. As a result, there are a total of 10 parameters that can be varied in the process of transforming the source form on the left to any given target form on the right.

Thus, with the O'Connor–Robertson program one can actually twist the 10 "knobs" independently, which corresponds to varying the 10 parameters in p and q until a particular target form is generated. Although Thompson used a wide variety of transformations, O'Connor and Robertson have discovered that most of his effects can be achieved by quadratic maps of the above sort. Consequently, by finding the parameters defining a particular quadratic map leading from one known species to another, we can hope to gain insight into the physical and evolutionary forces acting on different species. Alternately, we can also use the program to study intermediate forms like that of Figure 5.5, which have never before been observed and possibly never even existed. Such novel forms are interesting as objects that *might* exist sometime in the future. They are also of interest as objects of investigation to discover what features they have that prevented their appearance in the past.

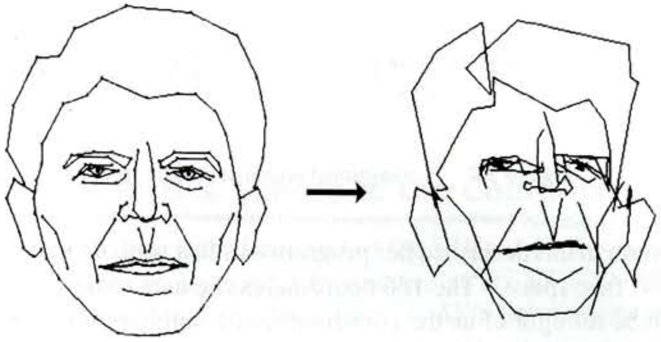


Figure 5.6 A computer caricature of Ronald Reagan.

The problem with these kinds of continuous deformations is that nothing essentially new ever turns up; by definition, there can be no *discontinuous* jumps from one species to an entirely different one via a sequence of continuous transformations. For this type of speciation to take place, we need *singularities* in the families of continuous transformations. Although there is no room to discuss the matter further here, let me note in passing that the mathematical theory of catastrophes was developed around 30 years ago to deal with precisely this situation. More information on this line of research is available in the volumes cited in the references.

One set of images used by Thompson in his book was taken from work on facial angles by the artist Albrecht Dürer. These experiments by Thompson on continuous distortions of such angles led to a wide variety of faces, calling to mind a program written a few years ago by Susan Brennan of the Hewlett-Packard Laboratories. Basically, her program is designed to continuously deform a given face in order to create a caricature of it. Serving to motivate the development of this program was the question of how humans so quickly recognize faces, even when only a few features are seen under adverse viewing conditions. Brennan hoped that her program could be used as a tool for investigating just which features people focus upon in this remarkable pattern-recognition process. The details of how the program works have been described elsewhere, but the principle is simplicity itself: Compare the target face with an "average" face, and then scale up those features that differ the most from the average face. An example of how the program caricaturizes former President Ronald Reagan is shown in Figure 5.6.

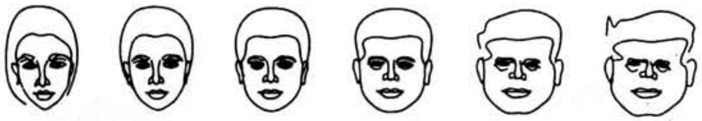


Figure 5.7 A transitional sequence in face space.

Brennan has described her program as a fast way of exploring what she calls "face space." The 186 coordinates she uses to describe a target face can be thought of as the coordinates of a single point in a space of dimension 186. Because every face is a point in this same space, any two faces can be connected by a straight line in face space, each point along the line representing proportional changes in every coordinate value. The distance between any two points then serves as a measure of how similar the faces are. With d'Arcy Thompson in mind, we can also ask the computer to generate a transitional sequence from one face to another. Such a transition sequence from Elizabeth Taylor as Cleopatra to John F. Kennedy is shown in Figure 5.7.

A key factor motivating the work of both d'Arcy Thompson and Susan Brennan is to uncover *invariants* of the transformations from one form to another. If these transformations have any biological or psychological content at all, presumably it lies in telling us what it is *exactly* that allows us to recognize the caricatured version of Ronald Reagan as Ronald Reagan and not, say, JFK, or what *exactly* are the characteristics shared by the fish species *Scarus sp.* and *Pomacanthus.* Only a deeper understanding of such invariants will enable us to unlock the complexity of living things, by showing us the features that remain unchanged as we pass from one species to another along a smooth, evolutionary pathway. As d'Arcy himself put it, "I know that in the study of material things number, order, and position are the threefold clue to exact knowledge; and that these three, in the mathematician's hands, furnish the first outlines for a sketch of the Universe."

From these considerations, I think it is plain to see that scientists like d'Arcy Thompson would have welcomed the computer as an ally in their quest to unlock the secrets of nature and humans. However, in order for complexity theory as embodied in these would-be worlds to have an impact in the real world of science, it will take more than the kind of empirical observations provided by the kinds of models we have been discussing in this volume. It will take a workable *theory* of complex

systems. Let us conclude our story with an account of what's needed to put such a theory in place.

Toward a Theory of the Complex

In chapter 3, we gave a brief account of the El Farol Problem, in which Irishmen in Santa Fe try to decide whether or not to go to the bar to listen to Irish music on Thursday evenings. Although I doubt seriously that any real Irishman like Brian Arthur would be deterred in the least from a visit to the local pub simply by the presence of others of like mind, let's continue the fantasy outlined in the El Farol Problem as a way of encapsulating a large fraction of the problems encountered in the would-be worlds considered in this book.

Recall that the problem faced by the Santa Fe Irishmen was to use their currently best rule to estimate how many music lovers would appear at El Farol in the coming week. On the basis of this prediction, each individual then chose to go to the bar or stay home, with the total number attending being reported to everyone in the following week. At that time, each Irishman revised his or her set of predictors, using the most accurate predictor to estimate the attendance in the coming week. And so it goes, week after week, until the Irishmen got tired of the music and/or the drinking or, what is far more likely (and actually happened), the band of Irish musicians moves on to greener pastures. It is my belief that the key components forming the El Farol Problem are exactly the key components in each and every one of the would-be worlds discussed in the last chapter, and that a decent mathematical formalism to describe and analyze the El Farol Problem would go a long way toward the creation of a viable theory of complex, adaptive systems. So let's look at what these key components are.

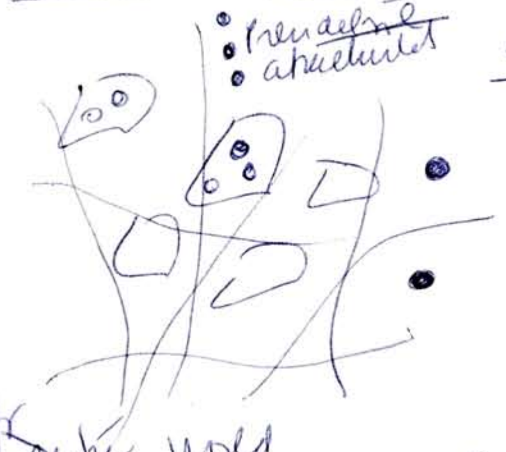
***Medium-Sized* Number of Agents.** In the El Farol Problem we have postulated 100 Irishmen, each of whom acts independently in deciding to go or not go to the bar on Thursday evening. In contrast to simple systems—like superpower conflicts, which tend to involve a small number of interacting agents—or large systems—like galaxies or containers of gas, which have a large enough collection of agents that we can use statistical means to study them—complex systems involve what we might call a medium-sized number of agents. Just like Goldilocks's porridge, which was not too hot and not too cold, complex systems have a number of agents that is not too small and not too big, but just right to create interesting patterns of behavior.

✱ **Intelligent and Adaptive.** Not only are there a medium number of agents, these agents are intelligent and adaptive. This means that they make decisions on the basis of rules, and that they are ready to modify the rules they use on the basis of new information that becomes available. Moreover, the agents are able to generate new rules that have never before been used, rather than being hemmed in by having to choose from a set of preselected rules for action. This means that an ecology of rules emerges, one that continues to evolve during the course of the process.

✱ **Local Information.** In our would-be worlds, no single agent has access to what *all* the other agents are doing. At most, each agent gets information from a relatively small subset of the set of agents, and processes this "local" information to come to a decision as to how he or she will act. In the El Farol Problem, for instance, the local information is as local as it can be, because each Irishman knows only what he or she is doing; none have information about the actions taken by any other agent in the system. This is an extreme case, however, and in most would-be worlds the agents are more like drivers in a transport system or traders in a market, each of whom has information about what a few of the other drivers or traders are doing.

So these are the components of almost all complex systems like the El Farol situation—a medium-sized number of intelligent, adaptive agents interacting on the basis of local information. At present, there appears to be no known mathematical structures within which we can comfortably accommodate a description of the El Farol Problem. This suggests a situation completely analogous to that faced by gamblers in the seventeenth century, who sought a rational way to divide the stakes in a game of dice when the game had to be terminated prematurely (probably by the appearance of the police or, perhaps, the gamblers' wives). The description and analysis of that very definite real-world problem led Fermat and Pascal to the creation of a mathematical formalism we now call probability theory. At present, complex-system theory still awaits its

Pascal and Fermat. The mathematical concepts and methods currently available were developed, by and large, to describe systems composed of material objects like planets and atoms. But as philosopher George Gilder has noted, "'The central event of the twentieth century is the overthrow of matter. In technology, economics, and the politics of nations, wealth in the form of physical resources is steadily declining in value and significance. The powers of mind are everywhere ascendant over the brute force of things.'" It is the development of a proper theory of complex systems that will be the capstone to this transition from the material to the informational.



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